

# EP214: Analog Signals and Systems

## Lecture Notes

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# 1 Complex Numbers Review

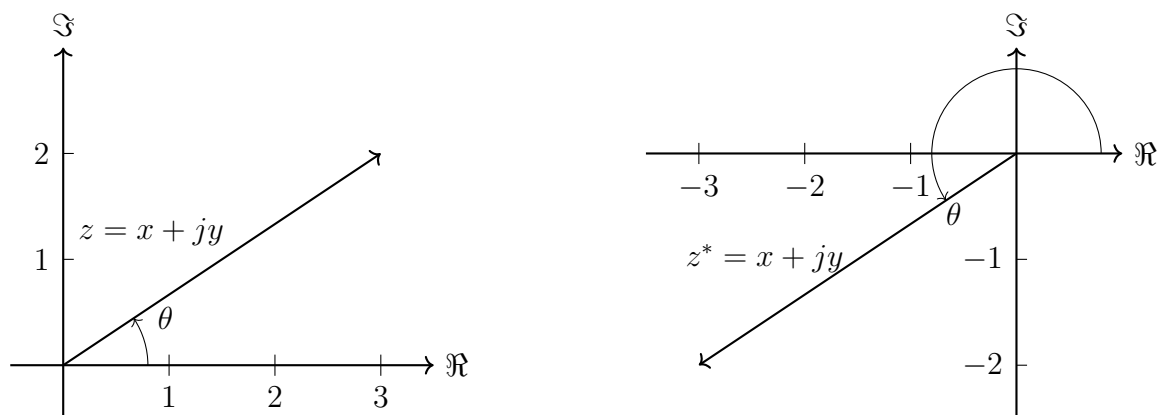
## 1.1 Complex Numbers

$z = x + jy$  is a complex number where  $x$  is real and  $jy$  is imaginary.  $x$  and  $y$  are both real numbers where  $j$  is the imaginary unit, defined by  $j^2 = -1$ . A complex number is a composite of real and imaginary numbers, where...

$\text{Re}(z) = x$  is the real part.

$\text{Im}(z) = y$  is the imaginary part.

## 1.2 Complex Plane



In the figures above we can see that we are able to decompose the real and imaginary components of each vector  $z$  through the following. Let  $M$  be the magnitude of the vector  $z$ .

$$x = M \cos(\theta)$$

$$y = M \sin(\theta)$$

Where  $\cos(\theta)$  is the real part and  $\sin(\theta)$  is the imaginary part. Thus,

$$z = x + jy = M(\cos(\theta) + j \sin(\theta))$$

To then find angle theta we can use trig identities to find the following.

$$\theta = \arctan\left(\frac{y}{x}\right)$$

**Example:** Find  $M$  and  $\theta$  as shown above for both.

$$z = x + jy \Rightarrow 3 + j2$$

$$M = \sqrt{x^2 + y^2} \Rightarrow \sqrt{9 + 4} \Rightarrow \sqrt{13}$$

$$\theta = \arctan\left(\frac{y}{x}\right) \Rightarrow \arctan\left(\frac{2}{3}\right) \Rightarrow 0.588 \text{ rad}$$

$$z = M(\cos(\theta) + j \sin(\theta)) \Rightarrow \sqrt{13}(\cos(0.588) + j \sin(0.588))$$

Given that the magnitudes of the real and imaginary axes are the same, to find the complex conjugate we simply need to add  $\pi$  to the angle to correct for the vector being in the third quadrant.

$$z = \sqrt{13}(\cos(3.729) + j \sin(3.729))$$

### 1.3 Inverse Tangent Identities

$$\tan^{-1}\left(\frac{-B}{A}\right) = -\tan^{-1}\left(\frac{B}{A}\right)$$

$$\tan^{-1}\left(\frac{B}{-A}\right) = \pi - \tan^{-1}\left(\frac{B}{A}\right)$$

### 1.4 Euler's Theorem

Given,

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

We can see that,

$$z = x + jy = M(\cos(\theta) + j \sin(\theta)) = Me^{j\theta}$$

Noting the following relationship.

$$e^{-j\theta} = \cos(-\theta) + j \sin(-\theta) = \cos(\theta) - j \sin(\theta)$$

### 1.5 Complex Conjugate

Every complex number has a complex conjugate where the imaginary component is negated. In otherwords, is reflected about the real-axis.

$$z = x + jy \Rightarrow z^* = x - jy$$

$$z = Me^{j\theta} \Rightarrow z^* = Me^{-j\theta}$$

### 1.6 Important Identities

$$zz^* = Me^{j\theta}Me^{-j\theta} = M^2$$

$$z + z^* = x + jy + x - jy = 2x$$

$$\text{Re}(z) = x \Rightarrow \frac{z + z^*}{2}$$

$$z - z^* = x + jy - (x - jy) = j2y$$

$$\text{Im}(z) = \frac{z - z^*}{j2}$$

## 2 Linear Systems

### 2.1 Introduction to Linear Systems

A system processes / modifies information from input signals to product output signals.

$$\text{Input} \longrightarrow \boxed{S} \longrightarrow \text{Output}.$$

The system has a set of rules to go from input to output.

We say that system  $S$  operates on input  $x$  to produce output  $y$ .

$$x \longrightarrow \boxed{S} \longrightarrow y.$$

$$sx = y \quad \underline{\text{or}} \quad x \xrightarrow{S} y.$$

There are two rules regarding linear systems:

1. Proportionality.

If  $sx = y$ , then  $s(cx) = c(sx) = cy$ . Where  $c$  is a constant.

2. Superposition

If  $sx_1 = y_1$  and  $sx_2 = y_2$ , then  $s(x_1 + x_2) = sx_1 + sx_2 = y_1 + y_2$

Putting these two rules together give us:

$$\begin{aligned} s(c_1x_1 + c_2x_2) &= c_1sx_1 + c_2sx_2 \\ &= c_1y_1 + c_2y_2 \end{aligned}$$

Consider the following,

Where  $x_n = \tilde{c}_n e^{j\omega_n t}$

$$\begin{aligned} x(t) &= \sum_{n=0}^{\infty} \tilde{c}_n e^{j\omega_n t} \\ &= x_0 + x_1 + x_2 + \dots \end{aligned}$$

$$\begin{aligned} \text{Then } y(t) &= \sum_{n=0}^{\infty} \tilde{c}_n s e^{j\omega_n t} \\ &= y_0 + y_1 + y_2 + \dots \end{aligned}$$

Where  $y_n = \tilde{c}_n s e^{j\omega_n t}$

Thus, any real signal can be expressed as

$$\Re \left[ \sum_{n=0}^{\infty} \tilde{c}_n e^{j\omega_n t} \right].$$

If  $s e^{j\omega t}$  is known,

$$e^{j\omega t} \longrightarrow \boxed{S} \xrightarrow{s e^{j\omega t}}$$

then the output of  $S$  is known for any given input.

## 2.2 Types of Linear Systems

### 2.2.1 Time Invariant and Time Variant Systems

#### Time Invariant

System parameters don't change with time.

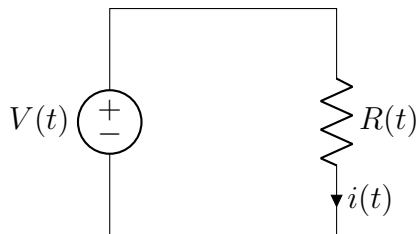
Output does not depend on when input is given.

$$\begin{aligned} x(t) &\rightarrow [S] \xrightarrow{y(t)} [\text{Time Delay}] \xrightarrow{y(t-a)} . \\ x(t) &\rightarrow [\text{Time Delay}] \xrightarrow{x(t-a)} [S] \xrightarrow{y(t-a)} . \end{aligned}$$

#### Time Variant

The systems behavior changes with time.

Consider the following circuit.



Input:  $V(t)$

Output:  $i(t) = \frac{V(t)}{R(t)}$

Where resistance,  $R(t)$  varies with time.

$$\begin{aligned} v(t) &\rightarrow [S] \rightarrow i(t) = \frac{v(t)}{r(t)} \rightarrow [\text{Time Delay}] \rightarrow \frac{v(t-a)}{r(t-a)} . \\ v(t) &\rightarrow [\text{Time Delay}] \rightarrow v(t-a) \rightarrow [S] \rightarrow \frac{v(t-a)}{r(t)} . \end{aligned}$$

As we can see, delaying the input and then applying the system gives us a different result then applying the system and then delaying.

Thus we can say that the output depends on when the input is given.

In EP214, we will be dealing exclusively with time invariant systems.

### 2.2.2 Dynamic and Instantaneous Systems

#### Dynamic

Output depends on previous input.

$$y(t) = \int_{-\infty}^t x(\tau) d\tau.$$

Instantaneous Output depends only on current input at time  $t$ .

$$y(t) = \frac{dx(t)}{dt}.$$

### 2.2.3 Causal and Non-Causal Systems

#### Causal

Output depends only on past and current input.

#### Non-Casual

Output depends on past, present, and future inputs.

## 2.2.4 Invertible and Non-Invertible Systems

### Invertible

Given the output, it is possible to figure out the input. It is possible to "undo" the result.

### Non-Invertible

Not all inputs have unique outputs.

$$y(t) = |x(t)|.$$

## 2.2.5 Stable and Unstable Systems

### Stable

Bounded inputs give bounded outputs.

### Unstable

Bounded inputs give unbounded outputs.

## 2.2.6 Mathematical Systems

Define  $D$  as a system that differentiates.

$$\xrightarrow{x(t)} \boxed{D} \xrightarrow{y(t)} .$$

$$Dx = \frac{dx}{dt} = \dot{x} = y.$$

An example being:  $x(t) = t^2 - 1$

$$\xrightarrow{x(t) = t^2 - 1} \boxed{D} \xrightarrow{y(t) = 2t} .$$

## 2.2.7 Inverse System

In this system  $S^{-1}$  undoes  $S$ .

$$\xrightarrow{x(t)} \boxed{S} \xrightarrow{y_1(t)} \boxed{S^{-1}} \xrightarrow{y_2(t) = x(t)} .$$

$$y_2(t) = s^{-1}y_1(t) = s^{-1}sx(t) = x(t).$$

An example being:  $D^{-1}x(t) = \int_0^t x(\tau) d\tau$

$$\xrightarrow{x(t)} \boxed{D^{-1}} \xrightarrow{y_1(t)} \boxed{D} \xrightarrow{y_2(t)} .$$

$D$  is the inverse of  $D^{-1}$ .

$$y_2(t) = \int_0^t y_1(\tau) d\tau = \int_0^t \frac{dx(\tau)}{d\tau} d\tau = x(\tau)|_0^t = x(t) - x(0).$$

$D^{-1}$  is the inverse of  $D$  up to a constant.

## 3 Laplace Transforms

### 3.1 Introduction to Laplace Transforms

The Laplace Transform is a mathematical tool used to solve time independent linear differential equations.

$$X(s) = \int_{-\infty}^{\infty} X(t)e^{-st} dt.$$

Where  $X(s)$  is the Laplace transform of  $X(t)$  and  $S$  is a complex number.

We say that:

$$X(s) = \mathcal{L}[x(t)]$$

$$x(t) = \mathcal{L}^{-1}[X(s)]$$

Where  $x(t)$  is the inverse Laplace transform of  $X(s)$ .

The Laplace transform is a linear operator, meaning it obeys proportionality and superposition.

$$\begin{aligned}\mathcal{L}[aX_1(t) + bX_2(t)] &= \int_{-\infty}^{\infty} [aX_1(t) + bX_2(t)]e^{-st} dt \\ &= a \int_{-\infty}^{\infty} X_1(t)e^{-st} dt + b \int_{-\infty}^{\infty} X_2(t)e^{-st} dt \\ &= a\mathcal{L}[X_1(t)] + b\mathcal{L}[X_2(t)]\end{aligned}$$

### 3.2 Exponential Signals

For an exponential function multiplied by the unit step function,

$$X(t) = e^{-at}u(t) = \begin{cases} e^{-at} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}.$$

Where  $a$  is a constant.  $X(s)$  is then,

$$\begin{aligned}X(s) &= \int_{-\infty}^{\infty} X(t)e^{-st} dt \\ &= \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st} dt \\ &= \int_0^{\infty} e^{-at}e^{-st} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \\ &= -\frac{1}{s+a}e^{-(s+a)t} \Big|_0^{\infty} \\ &= -\frac{1}{s+a}[e^{-(s+a)\cdot\infty} - e^{-(s+a)\cdot 0}] \\ &= \frac{1}{s+a}[1 - e^{-(s+a)\cdot\infty}]\end{aligned}$$

$X(s)$  is only finite for values of  $S$  where,  $e^{-(s+a)\cdot\infty} = 0$ .

$$\begin{aligned} s + a &= \Re[s + a] + j\Im[s + a] \\ e^{-(s+a)t} &= e^{-(\Re[s+a] + j\Im[s+a])t} \\ &= e^{-\Re[s+a]t} e^{-j\Im[s+a]t} \end{aligned}$$

Where the real component is the magnitude and the imaginary component is the complex phase. Then,

$$|e^{-(s+a)t}| = e^{-\Re[s+a]t}$$

If  $\Re[s + a] > 0$ , then

$$\lim_{t \rightarrow \infty} e^{-(s+a)t} = 0.$$

We will always assume  $\Re[s + a] > 0$ , therefore  $\Re[s] > -\Re[a]$ . Which implies,

$$X(s) = \frac{1}{s + a}.$$

$$\mathcal{L}[e^{-at}u(t)] = \frac{1}{s + a}.$$

### 3.3 Unit Step Function

$$\begin{aligned} \mathcal{L}[u(t)] &= \int_{-\infty}^{\infty} u(t)e^{-st} dt \\ &= \int_0^{\infty} e^{-st} dt \\ &= -\frac{1}{s}e^{-st} \Big|_0^{\infty} \\ &= -\frac{1}{s}[e^{-s\cdot\infty} - e^0] \\ &= -\frac{1}{s}[0 - 1] \\ &= \frac{1}{s} \end{aligned}$$

From this, if we assume  $t \geq 0$ , we see that

$$x(t) = 1 \xrightarrow{\mathcal{L}} X(s) = \frac{1}{s}.$$

Then, assume

$$\mathcal{L}[x(t)] = \mathcal{L}[x(t)u(t)].$$