EE241: Introduction to Electric Power Systems

Lecture Notes

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Contents

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1 Single Phase Circuits

1.1 Resistive Circuits

In resistive circuits, voltage and current are in phase with each other. As such, the angle between V and I is 0°. Thus, ϕ, the phase shift, is 0.
The power factor is calculated as the cosine of the phase shift between voltage and current. Thus, in a resistive circuit, the power factor is:

$$\cos\phi = \cos 0 = 1.$$

Where a power factor of 1 is called *unity*.
The voltage across and current through the resistor is calculated as shown below.

$$V_R = V\sin\omega t \Rightarrow V_R = \frac{V}{\sqrt{2}}\angle 0^\circ$$
$$I_R = I\sin\omega t \rightarrow I = \frac{I}{\sqrt{2}}\angle 0^\circ$$

1.2 Inductive Circuits

In inductive circuits, current lags voltage by 90°. As such, the angle between V and I is 90°. Thus, the phase shift, ϕ, is 90°.
The power factor is then:

$$\cos\phi = \cos 90 = 0 \text{ lagging.}$$

The voltage across and current through the inductor is then calculated as.

$$V_L = V\sin\omega t \Rightarrow V_R = \frac{V}{\sqrt{2}}\angle 0^\circ$$
$$I_L = I\sin\omega t \rightarrow I = \frac{I}{\sqrt{2}}\angle - 90^\circ$$

1.3 Capacitive Circuits

In capacitive circuits, current leads voltage by 90°. Thus, the phase angle, ϕ is 90°.
The power factor is then calculated as:

$$\cos\phi = \cos 90 = 0 \text{ leading.}$$

The voltage across and current though the capacitor is then:

$$V_C = V\sin\omega t \Rightarrow V_R = \frac{V}{\sqrt{2}}\angle 0^\circ$$
$$I_C = I\sin\omega t \rightarrow I = \frac{I}{\sqrt{2}}\angle + 90^\circ$$

1.4 Power Circuit Analysis

There are two methods in which we can analyze circuits.

Time Domain Method

- Applicable to both transient and steady-state circuit analysis.
- Very useful for transient analysis.
- Difficult method that often requires differentiation or integration of sinusoidal functions.

Phasor Method

- Only steady-state circuit analysis.
- An easy method.
- Sinusoidal functions are represented by magnitude (RMS) and phase angle.
- Differentiation/Integration is replaced by Multiplication/Division.

$$\text{Time Domain} \quad V \sin \omega t \longrightarrow \frac{V}{\sqrt{2}}\angle 0^\circ \quad \text{Phasor Domain.}$$

1.4.1 Example —— Phasor Method

Given two AC voltage sources in series, what is the total voltage supplied by the two sources where

$$V_1 = 28.28 \sin(\omega t)$$
$$V_2 = 14.14 \sin(\omega t + 45^\circ)$$

Converting the above to phasors, we need to convert from the peak value to the RMS value.
This will then be our magnitude, and the phase angle ϕ will be our angle.

$$V_1 = \frac{28.28}{\sqrt{2}}\angle 0^\circ$$
$$V_2 = \frac{14.14}{\sqrt{2}}\angle 45^\circ$$

When doing math with complex numbers, we add/subtract numbers in rectangular form and multiply/divide numbers in polar form.
As such, converting our phasors to rectangular form and adding gives us.

$$V = V_1 + V_2$$
$$V = 20(\cos 0 + j \sin 0) + 10(\cos 45 + j \sin 45)$$
$$V = 20 + j0 + 7.071 + j7.071$$
$$V = 27.071 + j7.071$$

Converting back into polar form gives us.

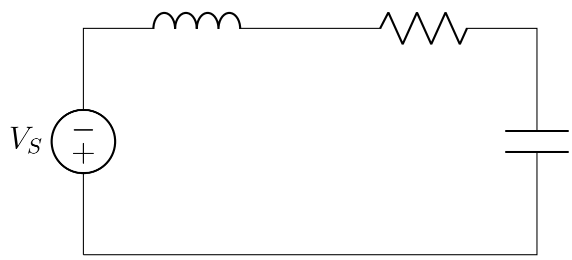
$$V = 27.979\angle 14.64^\circ.$$

To then get our total voltage in the time domain, we convert back to peak voltage from RMS voltage and express as a sinusoid.

$$V = 27.979\sqrt{2} \sin(\omega t + 14.64^\circ)$$
$$V = 39.57 \sin(\omega t + 14.64^\circ)$$

1.4.2 Example —— RLC Circuit

For the given circuit operating at 60Hz, find the total impedance, power factor, and current.



Where the value of circuit elements is as follows:

$V = 100\sin(\omega t + 30°)$

$R = 10\Omega$

$L = 26.525\text{mH}$

$C = 132.63\text{uF}$

To find the total impedance, we can first find the reactance of the inductor.

$$X_L = \omega L = (377\text{rad/s})(0.0265\text{H}) = 10\Omega.$$

Similarly, we can find the capacitive reactance.

$$X_C = \frac{1}{\omega C} = \frac{1}{(377\text{rad/s})(132.63 \times 10^{-6}\text{F})} = 20\Omega.$$

Because all of the circuit components are in series, the total impedance is given by:

$$\begin{aligned} Z &= R + jX_L - jX_C \\ &= 10 + j10 - j20 \\ &= 10 - j10 \\ &= 14.14\angle - 45° \end{aligned}$$

To then find the power factor, we need to take the cosine of the angle between the voltage and current.

$$\cos\phi = \cos -45° = 0.707 \text{ leading}.$$

Note that the power factor is leading because the phase angle is negative, indicating a capacitive load, which means that current leads voltage.
Lastly, to find the current, we can use Ohm's law.

$$I = \frac{V}{Z} = \frac{70.71\angle 30°}{14.14\angle - 45°} = \frac{70.71}{14.14}\angle 30° - (-45°) = 5\angle 75° A.$$

1.5 Real and Reactive Power

Real and reactive power have different properties.

Real Power (P)

Useful power.

Measured in Watts (W).

Capable of doing useful work.